Temporal considerations in collapse of WTC Towers

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Abstract: The circumstances leading to the collapse of the WTC Towers were described in numerous publications before but quantification of possible mechanisms published so far remains very limited. The basic observation is that columns of a 110-story building were weakened, over a relatively short segment of an upper part of the structure, to a degree where they were unable to support the building above them. As the upper part began to descend, successive buckling of columns caused flattening of the stories below. The process was presumably driven by the action of gravity until a complete destruction of the building. This article concentrates on progressive collapse of the core of the building. Several mechanisms are considered and quantified, to assess whether they offered a plausible explanation. One of the criteria used was whether the potential energy available was sufficient to cause the demolition in the assumed manner. The calculated duration of the event versus the available observation is regarded as the main criterion to qualify the postulated collapse mode. The details presented here are in reference to the North Tower. Some relationships presented here are also useful for a progressive collapse analysis of reinforced concrete structures.

Keywords: impact; shock; plasticity; progressive collapse; structural engineering; large deflections.

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Biographical notes: Gregory Szuladziński holds a Masters degree in Mechanical Engineering from Warsaw University of Technology in 1965 and a Doctoral degree in Structural Mechanics from the University of Southern California in 1973. From 1966 to 1980, he worked in the USA in the fields of aerospace, nuclear and shipbuilding industries. Since 1981, he has been working in Australia. His book entitled *Dynamics of Structures and Machinery. Problems and Solutions* was published in 1982. His second book, *Formulas for Structural and Mechanical Shock and Impact* (CRC Press) appeared worldwide in October 2009. He is also a Fellow of the Institute of Engineers Australia and member of ASME and ASCE.

1 Introduction

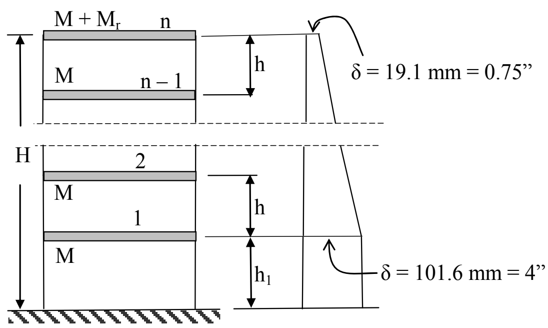
The geometrical, inertial and material data were provided in FEMA Report (2003) and summarised accordingly in Szuladzinski (2003). The most frequent explanation of the collapse mechanism of the buildings was a successive flattening of stories referred to as *pancaking*. The best known writer putting forward this hypothesis was Bažant, in Bažant and Verdure (2007) as well as in earlier papers. One of the discussion papers to Bažant and Verdure (2007) was Szuladziński (2008) where a detailed calculation of failure over the most heavily damaged story was presented. For the collapse to initiate the strength of the critical story had to be gradually degraded to just below the weight of the structure above it. The calculation showed that the energy lost in column squashing was larger than potential energy of gravity available for the process, thereby indicating the arrest of the motion. That calculation was done in reference to the vicinity of the critical story. In contrast, this paper takes a broader view of the entire building and its energy balance. In investigating the possibility of pancaking collapse all assumptions, often resulting from uncertainties in the available data, are made in a manner favourable to this collapse hypothesis.

The structural configuration of the building resulted in two independent vertical load paths, one along the outer shell (herein called *shell*) and the other along the core. The floors themselves were far too flexible to influence the distribution of the vertical floor load between the shell and the core, we can therefore assume that a uniform floor loading between those two paths would be divided between them on geometric basis and that the core can be treated independently from the outer shell with regard to the vertical load transmission. The tributary core area, 44.8 5m × 52.5 m, is enclosed inside a median line, drawn half-way between the core envelope and the shell contour. This approach allows one to isolate the vertical action of the core from that of the shell.

The geometry along the vertical direction is presented in Figure 1. The ground story ends at h_1 = 21.33 m and the entire building has H = 417 m. With 110 stories present, the typical story height is h = 3.63 m. The column wall thickness is 101.6 mm over the ground story and then it gradually decreases to 19.1 mm at the top level.

The resultant, average distributed mass over a typical floor was 505 kg/m^2, which is based on 300 kg/m^2 live load and representative of the tributary area of the core. When a relatively small column mass is added, after averaging it over the height of the building, the typical tributary floor mass becomes M = 1.402 kt (1 kt = 10^6 kg).

Figure 1 Geometry of North Tower core, mass distribution and wall thickness distribution



Note: Level n = 110 is the roof of the building.

Referring to Figure 1, the additional roof mass M_r = 3.324 kt has two components:

a the excess of the tributary roof mass over the typical floor mass, estimated as 1.18 kt

b part of the outer shell mass supported by the core after the impact and damage, 2.144 kt, as the effect of the weight redistribution, made possible by relatively rigid roof structure.

More details can be found in Szuladzinski (2003).

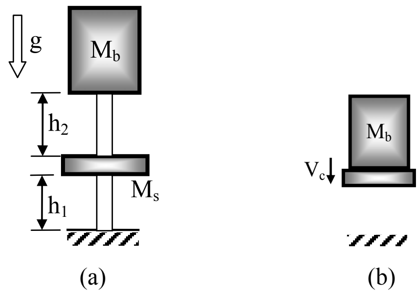
In a building with steel columns such as this one, changing the column section changes the vertical strength of the structure, but has a little effect on the structure mass. The main objective of this investigation is to assess the duration of collapse that took place, or the fall time. It will be convenient to occasionally assume the columns weaker than they really are and conduct the calculations accordingly. Such an assumption leads to decreasing vertical strength of the building and to the consequent shortening of the calculated fall time. In other words, this assumption yields a lower bound of the fall time.

2 Kinetics of plastic collision of two masses

Consider two masses, placed one above the other, as in Figure 2(a) and acted upon by gravity.

They are kept in position by frangible supports, which means supports collapsing under a moderate impulse. The upper support (under M_b) fails first, for some unspecified reason. The upper mass begins to accelerate and collides with the lower mass. As the collision is assumed to be plastic, both masses move in unison afterwards [Figure 2(b)] until they impact the ground.

Figure 2 Collision resulting from a collapse of frangible supports, (a) static condition prior to the event (b) motion after the lower support collapse following the collision



Based on simple kinematics and preservation of momentum, the following are obtained:

$$\mathrm{v}_0 = \sqrt{2gh_2} \tag{1}$$

$$\Delta\mathrm{v} = \mathrm{v}_0 - V_c = \frac{M_s \mathrm{v}_0}{M_b + M_s} \tag{2}$$

where v_0 is the velocity of M_b prior to impacting mass M_s, $V_c = \mathrm{v}_0 - \Delta\mathrm{v}$ is the velocity of assembly after the collision and $\Delta\mathrm{v}$ is the loss of velocity of M_b due to collision. The energy considerations yield the following:

$$\Delta E_k = \frac{1}{2} M * \mathrm{v}_0^2 = \frac{1}{2}\left(\frac{M_s}{M_s + M_b}\right) M_b \mathrm{v}_0^2 \tag{3}$$

$$E_g = \left(M_b h_2 + M_b h_1 + M_s h_1\right) g - \Delta E_k \tag{4}$$

$$v_f = \sqrt{2gh_1 + V_c^2} \tag{5}$$

where ΔE_k is kinetic energy loss due to the blocks colliding, E_g is the energy of the assembly

When impacting ground and v_f is the final impact velocity of the assembly. Finally, one has the impulse S applied to ground by the assembly and the event duration t_f (from the beginning to the end of motion), respectively as

$$S = (M_b + M_s) v_f \tag{6}$$

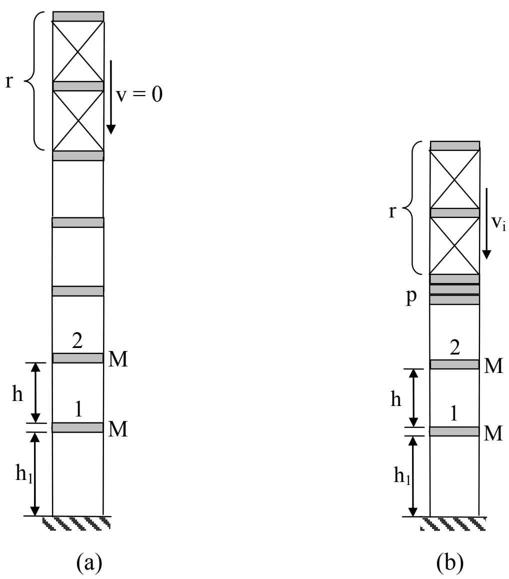
$$t_f = \sqrt{\frac{2h_2}{g}} + \frac{2h_1}{v_f + V_c} \tag{7}$$

A special case of the above is when $M_b = M_s = M$. Then $\Delta v = v_0/2$ and $\Delta E_k = M v_0^2/4$, which means one-half of the kinetic energy is lost in collision. If, on the other hand, $M_b \gg M_s$, then the loss of kinetic energy of M_b becomes close to $M_s v_0^2/2$.

3 Kinetics of sequential collisions, frangible or ductile columns

The previous section can be seen as description of a collapse of a two-story building with frangible columns. Here, the collapse of the centre core will be described initially ignoring loss of energy due to column deformation and introducing only the loss resulting from internal collisions, as appropriate for frangible columns (a *frangible* element is defined as absorbing a negligible amount of energy when crushed) (methodology explained in Szuladzinski, 2009).

Figure 3 Progressive collapse of a building, (a) intact structure (b) failure initiated at intermediate level with r intact stories moving down



Note: $r = 2$ is shown

In Figure 3, there are n stories, all identical except the ground story being taller. The motion, as depicted, is initiated by a story located below the r segments becoming weaker than necessary to support the weight applied to it. This is followed by the failure of columns in that intermediate story and the sequential 'pancaking' slabs below the intact part. Figure 3 shows $p = 3$ slabs pancaked and $p + r$ slabs in post-impact assembly.

When p slabs coalesce, it means p -1 collisions have taken place. The potential energy decrease resulting from the fall is:

$$U_g = \frac{2r+p}{2}(p-1)\,Mgh \tag{8}$$

This may be viewed as the gravitational, or potential energy absorbed, manifesting itself as the kinetic energy E_k. For a reason explained later, the collapse ends when all stories above the first become flattened. When the collapsing mass impacts the slab at h_1, then $n–r–1$ impacts have taken place and the same number of stories has been flattened. The potential energy absorbed by the falling masses is then

$$U_g = \frac{r+n}{2}(n-r-1)\,Mgh \tag{9}$$

Equation (3) tells us that the energy lost in collision of two masses is equal to the kinetic energy of the falling mass multiplied by the appropriate mass ratio. This comes out of the principle of momentum preservation and is true regardless of the value of r. If there is only a negligible loss of energy due to column resistance (a frangible column case) and $r = 0$, for example, then we note that after the first collision ($p = 2$), one-half of the potential energy absorbed is retained. A step-by-step procedure also shows that after the fourth collision ($p = 5$) the energy retained (E_{kn}) is 0.6 of U_g. Establishing the limit, to which the ratio of E_{kn} / U_g tends, is analytically complex, but rather easy numerically, using a spreadsheet. Such a procedure indicates that

$$E_{kn} = \frac{2}{3}U_g \quad \text{or} \quad \Delta E_{k1} \approx \frac{1}{3}U_g \tag{10}$$

is the limit, approached very closely when there is a large number of stories collapsing, $n > 100$.

In the above expression ΔE_{k1} is the energy lost in collisions.

The bottom end of the united group of slabs in Figure 3(b) could be called a *crushing front*, which moves down continuously until the motion is arrested. The thickness of individual slabs is shown for the accounting purpose only, as the constant shocks of collisions pulverise concrete and turn it into a cloud of dust. Some writers assume that a floor would squash to a finite thickness, say 10% of the original floor height. This make a lot of sense when materials, which are difficult to crush are involved, like steel or even wood for that matter. In case of WTC towers, the amount of steel in floor trusses was relatively small and, at the end, only a heap of mangled steel remained visible while concrete was dispersed. It is therefore reasonable to treat individual stories as fully compactable.

The moving mass, above the crushing front, consists of not only the slabs that have coalesced, but may also include the undamaged top part of the building. This is a variable mass, with a new floor being accreted after each downward movement by h. The entire reasoning so far was applicable to frangible columns.

When ductile columns are involved, there is a kinetic energy loss is due to compressive squashing. If Π stands for the energy lost by squashing of columns by h between the adjacent levels, then the total loss attributable to this cause is

$$\Delta E_{k2} = (n - r - 1)\,\Pi \tag{11}$$

The above value of Π is an average for the segment of building involved in collapse. The remaining kinetic energy of the falling mass is then

$$E_{kn} = U_g - \Delta E_{k1} - \Delta E_{k2} \tag{12}$$

When the entire kinetic energy is smaller, so is the energy lost in collisions:

$$\Delta E_{k1} \approx \frac{1}{3}\left(U_g - \Delta E_{k2}\right) \tag{13}$$

After substituting equation (11) and equation (13) into equation (12), one obtains

$$E_{kn} = \frac{2}{3}\left(U_g - \Delta E_{k2}\right) \tag{14}$$

The corresponding impact velocity v_n and the impact duration t are:

$$v_n = \sqrt{\frac{2E_{kn}}{nM}} \tag{15}$$

$$t \approx \frac{2(n - r - 1)\,h}{v_n} \tag{16}$$

Equation (16) is true for constant acceleration only. It is not exactly true for a variable mass problem at hand, but should be a good approximation if there is proportionality between the force applied to the crushing front (the force applied from below and representing the building strength) and the downward moving mass (this aspect will be discussed later). There was some evidence of a constant-acceleration process taking place during collapse, as described in Appendix B.

One of the necessary conditions for the collapse to continue is the positive value of the remaining kinetic energy E_{kn}. This holds not only for the target level (level 1) of the building, but also for the intermediate levels.

4 Estimate of resistance to squashing and absorbed energy

Consider a story that has vertical strength of P_u. In some codes of practice, this is called the *ultimate-limit* force. The design force, on the other hand, is the weight W of the building above the story in question. The ratio of the two is the factor of safety, $f = P_u / W$. One can therefore say that the initial resistance of a story is

$$P_u = fW \tag{17}$$

The safety factor is not an explicit variable used in ultimate design of buildings. When defined as above, it has several components. One of them is the multiple of the design load employed, which varies depending on circumstances, but it is, effectively, never less than 1.5. Another part of f is capacity reduction factor, which is another safety measure usually built into a design and has often the magnitude of about 0.8. When the design loads are increased and the estimated strength decreased, the resultant factor becomes $f = 1.5/0.8 = 1.875 \approx 1.9$. This will be used as the safety factor in the calculations to follow.

The squashing process reduces the kinetic energy of the falling building. If a story of height h is flattened, the average column resistance of some P_{av} is overcome in the process. It was calculated in the Appendix A that $P_{av} \approx 0.4\ P_u$ for the selected section. Consequently, this value will be used in the estimates below. The reduction of kinetic energy becomes

$$\Delta E_{ks} = P_{av} h = 0.4 P_u h = 0.4\,(1.9Wh) = 0.76Wh = \Pi \tag{18}$$

Usually, buildings are so constructed that the factor of safety is about constant along the height. The weight W increases linearly as we move down the building, provided all stories have the same mass. So does the strength as well as the kinetic energy loss, described by equation (18). When flattening is anticipated for a certain vertical segment of the building, the average Π can be used because of linearity. What is described here could be called a *gravity-resisting* building model.

There are two major deviations from such an idealised picture. The first is the reality of design, which dictates that the column section changes in final increments, which means that several stories will have the same strength. The lowest story of the constant-strength segment will have the prescribed safety factor while the highest one will be oversized, in relative terms (the change of strength at junction of the segments along the length may cause the downward motion of the crushing wave to be arrested.)

The second deviation is the fact that although the building may be under the influence of gravity only at some point in time the lateral design forces, while not present, increase sizes of structural elements. Neglecting this factor and the previous one results fall time.

Another minor deviation is initially damaged floors will be weaker and absorb less energy than the surrounding floors, but because their number is small, the difference can be ignored in the over-all energy calculations. In spite of all drawbacks, the gravity-resisting model is a very convenient computational tool. All the differences between the model and the real building produce a lower bound of the fall time.

The upper r stories of a building remain intact as long as their strength is not exceeded by impact forces.

If the over-all thickness distribution is as shown in Figure 1 and the stepping of thickness is in increments of 6.4mm (0.25”), then we have 13 distinct segments between level 1 and 110. A segment with thickness of 25.4 mm will stretch between level 93 and 101, which covers our area of interest.

5 Some possible patterns of collapse

The aircraft impact, assumed here to affect the floor between levels 95 and 96 to the greatest extent, caused much structural damage and eruption of fires. The direct and indirect result of those fires was such that the columns of that story were weakened to a degree where they were unable to support the structure above them. As the upper part began to descend, successive stories below underwent buckling of columns and flattening. This is the description of the pancaking mode of failure.

The downward motion starts from rest, with only a small excess of the weight of the upper part over the strength of the structure below. If, at any computation or simulation the initial velocity of the upper part is assumed to be non-zero, this quantity must be small. Taking that initial velocity as corresponding to a one-story free fall is unrealistic, although it was employed before, (Bažant and Verdure, 2007). Such an approach suggests a sudden vanishing of the entire floor, a physically inadmissible event a long time after the impact (treating the columns as frangible also implies vanishing of the columns below impact location, but it is used here only to obtain one bounding value of the result).

As stated above, this presentation is concerned with the downward motion of the core. Applying the same approach to the shell does not seem appropriate. Those columns formed assemblies, or 'trees'. The trees were connected by bolts, which often failed under secondary lateral deformation of the shell. It appears from the photographs and the video records that shell columns had typically undergone random deformation rather than axial squashing. In early stages of the event, some trees are seen as just falling off the building with little visible deformation suffered.

In quantifying the destruction of the building, one should first use equation (9) to equation (16) to find the parameters for the part below the damage zone and follow with the second phase, the squashing of the upper part. Only a small difference results in the calculated time if instead the entire event is treated as a top-initiated failure by putting $r = 0$.

The following collapse patterns will be considered using the methodology outlined above:

Pattern 1 The core collapses while structural interactions between parts of the building remain unchanged throughout the process.

Pattern 2 After a short initial phase the shell is damaged enough to become inactive. The entire weight of the shell of the upper part is transferred, via the roof structure, to the core. This is a rather pessimistic assessment of structural capacity remaining in the shell.

Pattern 3 The mass distribution is unchanged and the same as in pattern 1. The core columns become very weak, capable to resist only 1/10th of their design capability. This is to reflect a possibility of the loss of lateral supports because of the detachment of floors. This is more pessimistic than most engineers would anticipate, but it is interesting to see how such a dramatic decrease of structural capacity affects the fall time.

The lowest segment of the structure remained standing after the collapse, although the height of what remained was variable over the building footprint. The approach taken here was to treat the collapse as terminating at level 1, i.e., the top of the ground story. This is the reason why the energy balance equations presented below were set up on the collapse of n-1 stories in an n-story building. The examples of the three collapse patterns postulated above will now be presented.

5.1 Collapse, pattern 1

n = 110, number of stories in building

r = 14, number of stories between level 96 and 110, travelling initially undeformed

$$U_g = \frac{r+n}{2}(n-r-1)\ Mgh = \frac{14+110}{2}(110-14-1)\ Mgh = 5{,}890\ Mgh$$

is the potential energy absorbed, per equation (9). The additional roof mass M_r travels $H - h_1 = 417 - 21.33 = 395.67$ m. The total energy absorbed is therefore

$$\begin{aligned} U_{gt} &= 5{,}890\ Mgh + M_r g\left(H - h_1\right) \\ &= 5{,}890 \times 1.402 \times 10^6 \times 9.81 \times 3.63 + 3.324 \times 10^6 \times 9.81 \times 395.67 \\ &= 294.06 \times 10^9 + 12.9 \times 10^9 \\ &= 306.96 \times 10^9\ \mathrm{N-m} \end{aligned}$$

The average level, between 1 and 96 is ~ 48. The design mass above this level is (110 – 48) M = 62M plus the initial, additional mass of 1.18 kt, with a total of

$$M_m = 62 \times 1.402 + 1.18 = 88.104\ \text{kt.}$$

This gives a total weight above the mid-level of

$$W = M_m g = 88.104 \times 10^6 \times 9.81 = 864.3 \times 10^6\ \mathrm{N}$$

Energy loss due deformation, per story, equation (18):

$$\Pi = 0.76Wh = 0.76 \times 864.3 \times 10^6 \times 3.63 = 2{,}384 \times 10^6\ \mathrm{N-m}$$

The total, per equation (11):

$$\Delta E_{k2} = (n-r-1)\ \Pi = (110-14-1)\ 2{,}384 \times 10^6 = 226.5 \times 10^9\ \mathrm{N-m}$$

The remaining kinetic energy is, equation (14):

$$E_{km} = \frac{2}{3}\left(U_{gt} - \Delta E_{k2}\right) = \frac{2}{3}(306.96 - 226.5) \times 10^9 = 53.64\ \ 10^9\ \mathrm{N-m}$$

The effective core mass M_c is used in place of nM in equation (15):

$$M_c = 109 \times 1.402 \times 10^6 + 3.324 \times 10^6 = 156.14 \times 10^6\ \mathrm{kg}$$

Employing equation (15) and equation (16):

$$\mathrm{v}_n = \sqrt{\frac{2E_{kn}}{M_c}} = \sqrt{\frac{2 \times 53.64 \times 10^9}{156.14 \times 10^6}} = 26.21\ \mathrm{m/s}$$

$$t = \frac{2(n-1)h}{\mathrm{v}_n} = \frac{2(110-1)3.63}{26.21} = 30.19\ \mathrm{s}$$

The time needed for the crushing process of the upper part of the building would still have to be added to the above estimate.

5.2 Collapse, pattern 2

The approach is essentially the same as before, except for the extra roof mass becoming much bigger. The tributary mass of the outer part of the building (outside the median line mentioned) above is estimated to be 1.3178 kt per floor. If 14 floors become suspended off the roof structure, then the extra roof mass becomes

$$M_{r1} = 14 \times 1.3178 = 2.0 = 20.449 \text{ kt}$$

The above includes 2 kt as the excess of the roof mass over the typical floor mass. The new potential energy absorbed is

$$\begin{aligned} U_{gt1} &= 5{,}890\ Mgh + M_{r1} g \left(H - h_1\right) \\ &= 5{,}890 \times 1.402 \times 10^6 \times 9.81 \times 3.63 + 20.449 \times 10^6 \times 9.81 \times 395.67 \\ &= 294.06 \times 10^9 + 79.37 \times 10^9 \\ &= 373.44 \times 10^9 \,\text{N} - \text{m} \end{aligned}$$

ΔE_{k2} is as in pattern 1, therefore the remaining kinetic energy, per equation (12) is:

$$E_{kn} = \frac{2}{3}\left(U_{gt} - \Delta E_{k2}\right) = \frac{2}{3}(373.44 - 226.5) \times 10^9 = 97.96\ \ 10^9 \,\text{N} - \text{m}$$

The effective core mass is now

$$M_c = 109 \times 1.402 \times 10^6 + 20.449 \times 10^6 = 173.27 \times 10^6 \,\text{kg}$$

Acting as before obtain

$$\text{v}_n = \sqrt{\frac{2E_{kn}}{M_c}} = \sqrt{\frac{2 \times 97.96 \times 10^9}{173.27 \times 10^6}} = 33.63 \text{ m/s}$$

$$t = \frac{2(n-1)h}{\text{v}_n} = \frac{2(110-1)3.63}{33.63} = 23.53 \text{ s}$$

5.3 Collapse, pattern 3

This time we use only 1/10th of strength of columns, with the top part mass the same as in pattern 1. This is a continuous process, from top to the target level. The potential energy absorbed is obtained from equation (9) with $r = 0$:

$$U_g = \frac{n}{2}(n-1)\ Mgh = \frac{110}{2}(110-1)\ Mgh = 5{,}995\ Mgh$$

Including the additional roof mass M_r gives the total energy absorbed:

$$U_{gt} = 5{,}995\; Mgh + M_r g\left(H - h_1\right)$$
$$= 5{,}995 \times 1.402 \times 10^6 \times 9.81 \times 3.63 + 3.324 \times 10^6 \times 9.81 \times 395.67$$
$$= 299.3 \times 10^9 + 12.9 \times 10^9 = 312.2 \times 10^9\,\mathrm{N-m}$$

Using only 1/10th of Π employed before:

$$\Delta E_{k2} = (n-1)\,\Pi = (110-1)\;238.4 \times 10^6 = 25.99 \times 10^9\,\mathrm{N-m}$$

$$E_{kn} = \frac{2}{3}\left(U_{gt} - \Delta E_{k2}\right) = \frac{2}{3}(312.2 - 25.99) \times 10^9 = 190.81 \times 10^9\,\mathrm{N-m}$$

$$\mathrm{v}_n = \sqrt{\frac{2E_{kn}}{M_c}} = \sqrt{\frac{2 \times 190.81 \times 10^9}{156.14 \times 10^6}} = 49.44\ \mathrm{m/s}$$

$$t = \frac{2(n-1)h}{\mathrm{v}_n} = \frac{2(110-1)3.63}{49.44} = 16.01\,\mathrm{s}$$

6 Fall time summary

According to various observations, the collapse time of the North Tower was in the range from 9 s to 15 s. There seems to be only one certain source of measured time, a seismographic record from a fairly distant station, as quoted in *World Trade Center Building Performance Study* (2003). The strong earth tremor lasted 8s, but can be merely regarded as a lower bound of the collapse duration. Some time would be needed for the ground vibrations to build up to a high level and an additional interval, near the end of the process, could be anticipated to be a weakening signal.

As assumed in this work, the effective fall height is $H - h_1 = 395.67$ m. Simple kinetics shows that the fall time from that height, under gravity alone and without air resistance is 8.98 s.

This is the free fall time of the roof if there were nothing between the roof and level 1. The velocity of impact against the base is then 88.09 m/s.

When no column resistance is involved, but there are sequential collisions between the floor slabs, the peak impact velocity v_m is estimated by noting that one-third of potential energy is lost by those collisions. We have

$$\frac{1}{2} M_c \upsilon_m^2 = \frac{2}{3} U_g \quad or \quad \mathrm{v}_m^2 = \frac{4}{3}\frac{U_g}{M_c} \tag{19}$$

where M_c is the (adjusted) core mass. Taking the numbers from the last example, one has the adjusted potential energy $U_{gt} = 312.2 \times 10^9$ N-m and $M_c = 156.14 \times 10^6$ kg. Substituting into equation (19) gives us $\mathrm{v}_m = 51.63$ m/s. The duration of fall can be easily found if the motion is approximated by assuming a constant acceleration. It this case velocity changes linearly and we can write

$$H - h_1 = \mathrm{v}_m t_s / 2$$

where time t_s is the only unknown. Substituting, one obtains t_s = 15.33 s as the duration of fall.

In context of our problem, this is the lowest time limit, corresponding to frangible columns. Such a collapse can be imagined as accompanied by instant failure of all columns in the floor being impacted from above. There is no energy absorbed by columns in such a model. The associated average acceleration a can be found by writing an expression for the distance travelled, $H - h_1$. This gives us

$$a = 2(H - h_1)/t_s^2$$

or

$$a = 3.37 \text{ m/s}^2,$$

which is a fraction of acceleration of gravity.

Pattern 3 did include the complete collapse sequence and had duration not too different from the above, namely t_s = 16.01s. This is another way of saying that if columns are assumed to be weakened to a fraction of the original value, they help very little in slowing the fall.

In the analysis of Patterns 1 and 2 the fall times obtained were 30.19 s and 23.53 s, respectively. The columns were assumed to have a desired safety factor against gravity. This was a gravity-resisting model, which, as said before, results in a lower bound of the fall time. Also, the additional time needed to squash the upper part of the building was ignored.

One other way of estimating the duration would be to use a constant acceleration, as quoted in Appendix B, as applicable to the entire path. This leads to the fall time of 10.67 s. It is likely that the acceleration would decrease for lower parts of the building, which will result in a longer fall time.

7 Conclusions and discussion

In current practice of the ultimate state design, the factor of safety f is not explicitly used, but it may be inferred from the multiplying factors applied to design loads. The value of f = 1.9 used above is minimal; it is likely that in most locations f will substantially exceed this. The larger the f, the larger the resistance offered by the collapsing structure and the longer time the fall time.

In all examples, the value of f was based on the gravity-resisting model. Consequently, our procedure artificially decreased the energy absorbed by columns or shortens the fall time.

The average crushing force of a compressed column was found to be about 0.4 of a nominal plastic strength of a cross section. Consequently, the factor of 0.4 was used in the absorbed energy calculations. Note, however, that this reduction factor strongly depends on the wall thickness in any particular case.

The safety factor against gravity was underestimated, partly because the FEMA information was incomplete and a choice was made to use a minimum f. A more realistic number can be determined by comparing the weight W above level 96 with the core column strength at that level. The former can be obtained from Pattern 1 calculations replacing 62 stories with 14. The total section area is found as 48*A*, where *A* is determined in Appendix A. When this is multiplied by a reasonable crushing stress, about 250 MPa (one-half of plastic strength) the total crushing strength P_u is found. The latter indicates $f = P_u / W = 3.7$. The reason for such a large *f*, relative to what is needed for gravity only, is associated with the fact that a hurricane-strength wind of 100 mph (161 km/h) was the most dominating load. (While the core only moderately participates in over-all bending resistance, the core and the shell are on equal footing with regard to resisting lateral forces.) If such a large safety factor were used in the calculation, the result would be the arrest of the downward motion.

Note that the above calculations, which aimed to determine the lower bound of the fall time, were, in fact, favourable to the pancaking hypothesis. One other factors slowing the downward motion was not included; the resistance of air volume being compressed in front of the crushing wave.

The analysis of pattern 3 was to merely assess the effect of columns becoming unreasonably weak, inconsistent with their properties and their presumed action during the pancaking mode. How a weakening of columns to 1/10th of the nominal strength could transpire is rather hard to imagine. There is a possibility of the floors being stripped off the core columns, but this also removes the associated gravity load. Also, if the core could remain standing, over a substantial part of the building height, with much of the weight removed, how would it topple? A sideways fall of a part of the core (cantilever type buckling) becomes a natural failure mode then, but this would be against physical evidence of the event.

In summary, the pancaking mode is not a realistic proposition, as the calculated fall time becomes much too long. The only way to approach the observed duration is to assume pattern 3, quite an unrealistic proposition. This is not to say that the mode could not last only during a small part of the collapse, followed by a later arrest and a change of the collapse mode.

The subject of a progressive collapse of a concrete building comes naturally, as it is related to column frangibility. A concrete column can undergo only a very small axial shortening prior to collapse. The energy absorption due to crushing is miniscule compared with a steel column of the same strength. Therefore, concrete columns may be seen as resembling the ideally frangible elements.

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Appendix A

Column resistance in large-deflection compression

One of the methods to assess the resistance offered by a column past the elastic range is to treat it as a three-hinge mechanism as done by Bažant and Verdure (2007). The peak resistance is at the outset, when the column is (nearly) straight and is equal to the Euler buckling load P_{cr} moderated by plasticity of the column material. The resisting force decreases with deflection until the two arms of the mechanism become horizontal. The force is then a small fraction of its initial value.

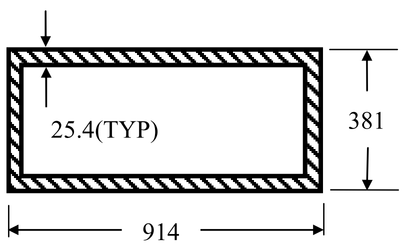
This approach was also used by Szuladziński (2008), but only in the first phase of deformation, up to the rotation angle of 75°, corresponding to axial shortening of 0.445 L, where L is the total column length. At that point the column resistance would reach its minimum of about $0.25P_{cr}$ and would increase thereafter due to contact between strongly deformed element of the column surfaces giving rise to new resisting mechanisms.

At that time it was only an intuitive approach that leads to postulating such a strengthening. A finite-element simulation was conducted to evaluate this effect, (Analytical Service Pty Ltd., 2008). A three-hinge deformation pattern developed was similar to one postulated before. The minimum resistance of $0.28P_{\rm cr}$ was attained at $0.38L$, earlier than expected. The resisting force was increasing with the increase of vertical deflection from then on.

A beam-like column buckling related to the above deflections is possible for a relatively slender beam. If, by over-all proportions, the beam is stout but its walls are rather thin, a local wall buckling leading to so-called harmonica mode will appear. A combination of both modes is possible, too.

The most likely, simplified section of a core column in the critical zone, inferred from FEMA reports appears in Figure A1.

Figure A1 Column section under consideration



The column material is A36 steel. Its nominal properties are: F_y = 36 ksi = 248 MPa (yield);

$$F_u = 60 \text{ ksi} = 414 \text{ MPa (ultimate strength)}$$

and

$$\varepsilon_u = 0.21 \text{ (ultimate strain).}$$

The multiplying factors were used to allow for statistical distribution (a difference between the minimum guaranteed and the average/expected properties) as well as the strain rate effect. The final values used were $F_y = 372$ MPa and $F_u = 501$ MPa.

The section area is $A = 63{,}205$ mm^2. The column is of low slenderness (with average $L = 3.63$ m), therefore one would expect the over-all buckling load to correspond to stress of just below F_y, or $P_{cr1} = AF_y = 23.51 \times 10^6$ N. This does not include a wall buckling effect. On the other hand, the local harmonica mode is associated with the following, estimated buckling load:

$$P_{cr2} = 12.16\left(\frac{c}{h}\right)^{0.37} h^2 F_y \tag{A1}$$

according to Szuladzinski (2009). Using the median-length dimensions per Figure A1, the average side length $c = 622.1$ mm and thickness $h = 25.4$ mm, therefore $P_{cr2} = 9.53 \times 10^6$ N. This is substantially smaller load than the previously calculated value, which suggests that the harmonica mode will dominate.

To obtain more accurate figures a FEA simulation of crushing was conducted using LS Dyna code (2005) with a model built of shell elements. The lateral movement during buckling is more likely to eventuate along the short sides of the cross-section, therefore symmetry was used to simplify the problem by constructing a half-model (this is the reason for the factor of 2 appearing in the calculations below.)

Figure A2 Early-stage deformation pattern (see online version for colours)

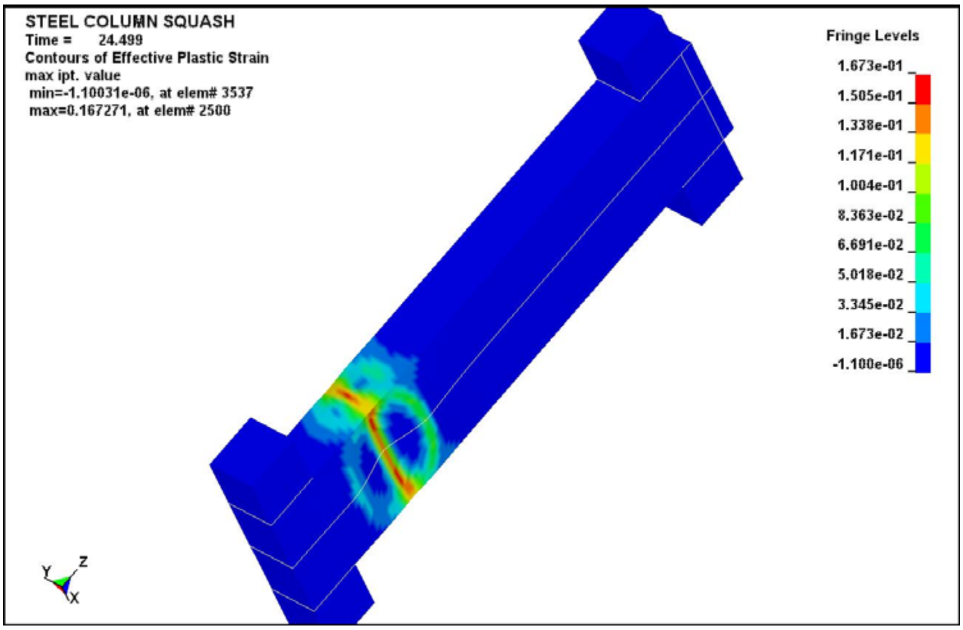
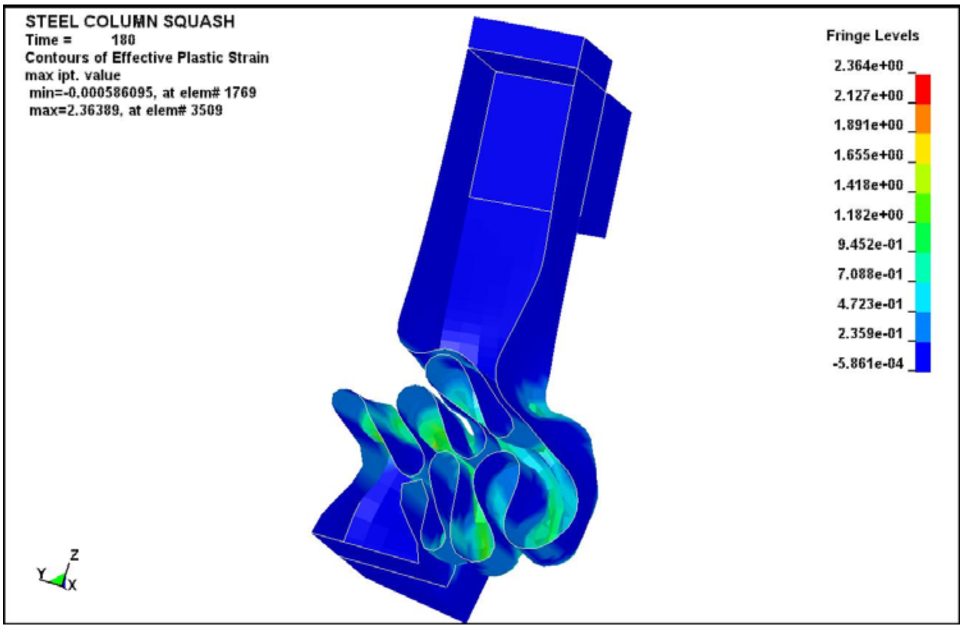
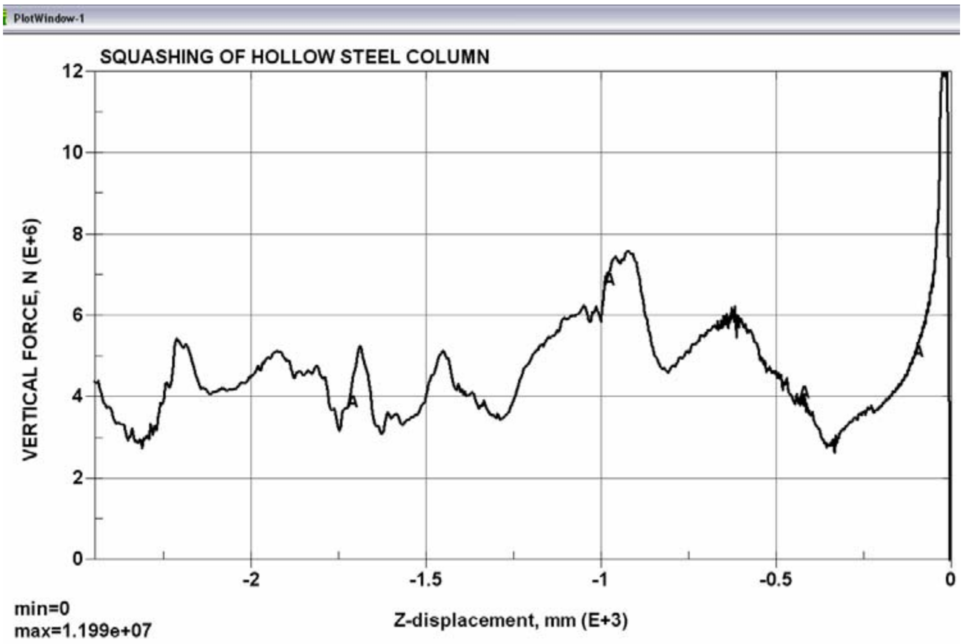


Figure A3 Advanced-stage deformation pattern (see online version for colours)



Note: This is a half-model.

Figure A4 Compressive resistance as a function of axial displacement (see online version for colours)



Note: Displacement growing to the left.

The rotations of ends were partially allowed by attaching those ends to structural elements similar to the column itself. The vertical displacement was excluded at the bottom end. A tributary mass was attached to the upper end to substitute for the portion of the building supported by the column. Applying a sudden acceleration to that mass, with a magnitude substantially larger than the column strength, was used as a means for dynamic crushing. The initial and the advanced deformation patterns are shown in Figure A2 and Figure A3.

The resistance-displacement history is given in Figure A4. The initial peak of $2 \times 11.99 \times 10^6 \approx 24 \times 10^6$ N corresponds to the plastic strength increased by dynamic action, i.e., lateral inertia of column elements producing slightly increased resistance. The remainder of the plot reflects creation and collapse of successive wrinkles. A theoretical estimate by means of equation (A1), namely $P_{cr2}/2 = 4.77 \times 10^6$ N is somewhat below the typical peaks in Figure A4.

The simulation was carried out until the peak axial displacement reached 2,447 mm, or 67% of column length. If the plot in Figure A4 is regarded as a deformation pattern representative of the entire squashing process, then the integration $\int P(t)\, du$, where u is the axial displacement gives the average resisting force of

$$P_{av} = 2 \times 4.67 \times 10^6 \,\mathrm{N} = 9.34 \times 10^6 \,\mathrm{N} \tag{A2}$$

If the nominal strength of the column is taken as the yield load, $P_{cr1} = 23.51 \times 10^6$ N, then we have

$$P_{av} / P_{cr1} = 0.397 \approx 0.4 \tag{A3}$$

One should note here that as deflection progresses and the possibility of further folds forming in the wall decreases, the resistance grows, in fact it will grow quite rapidly when approaching ‘bottom-out’ condition. Yet, the benefit from such a strong growth is limited for a column, which is only one element in a vertical assembly. The buckling capability of columns above and below will make it impossible for a very high resisting force to develop.

Appendix B

Some observable effects

There is a video of the initial phase of collapse of the North Tower (WCT1) taken by Etienne Sauret, a French filmmaker. That video was processed by MacQueen and Szamboti (2009) to quantify the motion of the roof. When velocity is plotted as a function of time, the resulting relationship is essentially linear with a small, cyclic oscillation superposed on it. The expression, at which the writers arrived, was v = 22.81 t, where v ~ ft/s and t ~ s. The acceleration involved is equivalent to 6.952 m/s^2 or about 0.71 g. This is one of several instances where a constant acceleration of a collapsing building was observed.

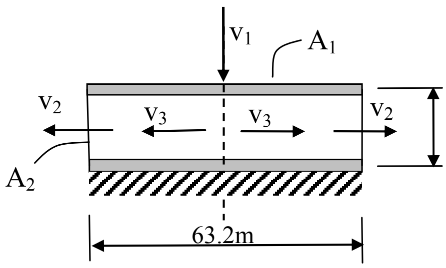
An interesting aspect of the film is the initial manner of collapse. There was a floor, which could be called a ‘line of fire’, a level just above which the internal fire was more visible than elsewhere. From a certain instant of time, associated with the beginning of downward movement of the roof, the upper part of the building seems to gradually crash against the line of fire. Interesting as this observation is, one should not read too much into it. If the collapse was initiated at the core part of the building, the crushing of the shell described above may, for example, be entirely consistent with the intact part of the core, above the line of fire, descending against the core below. The falling core would pull the floors down and those floors, acting then as tensile elements would in turn pull some shell columns inward.

When a storey is flattened, air is being pushed out as the adjacent floors come close together. A simplified scheme is shown in Figure B1, where v_1 is the speed of downward moving floor and v_2 is the side movement velocity of the expelled air. Based on preservation of volume, one can find that the relation between the velocities is

$$\upsilon_2 = \frac{A_1}{A_2}\upsilon_1 = \frac{63.2^2}{4 \times 62.2 \times 3.63} = 4.35\upsilon_1 \quad \text{(A4)}$$

This means that for a well-developed collapse, with v_1 = 30m/s for example, we have an outflow speed of v_2 = 130.5 m/s. But this is only an average along the height, with a large variation expected. Not only that; one can also expect variations along the perimeter due to unavoidable non-uniformity of conditions in a collapsing building. A variety of contents over the floor area would only increase the non-uniformity of flow around the perimeter. The peak local velocity is likely to much exceed what was calculated above.

Figure B1 The lower floor fixed



Note: The upper floor is suddenly acquiring velocity v_1 and maintaining it until in contact with lower floor.

Another simple estimate of an outflow velocity is provided by the observation that during time needed by the upper slab to collide with the lower one a particle of air located near the axis of the building will have to traverse the distance of s = 63.2/2 = 31.6 m. The time available is t = 3.63/v_1 = 0.121s. Therefore, the time-average v_3 = s/t = 31.6/0.121 = 261.2 m/s. After noting that at the outset v_3 = 0, one can expect a much higher peak velocity at the outer wall. When the spatial non-uniformities mentioned previously are considered, a peak value of ~2 × 261.2 = 522 m/s may be reasonably expected (some possible leakage through staircases is ignored).

When air is driven with the velocity exceeding the speed of sound (about 340 m/s under normal conditions) shock waves arise, which are associated with sounds resembling explosions. The reason for mentioning this effect is so-called conspiracy theory, which says that the collapse of WTC buildings was associated with numerous internal explosions. While this author does not take stand on the issue, it is appropriate to mention a mechanistic explanation of explosion-like sounds.

There is another interesting visual effect of explosive puffs emanating from the building and appearing well below the advancing crushing wave. In line with the previous explanation, those puffs would be the sign of a story in course of destruction. The reason for such a 'premature damage' is related to the safety factors involved, as presented before. In every constant-strength segment the lowest floor is the weakest one and can therefore fail ahead of the floors above as a consequence of repeated impacts transmitted along the height. In Szuladzinski (2003), there is a description of a simulation of the core collapse, which mentions and illustrates this effect (while that simulation had some imperfections, this writer expects every such analysis, better or worse, to show such an effect).